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FERMION GENERATIONS FROM THE STRONGLY COUPLED HIGGS SECTOR
(A Variation on a theme of Veltman)

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ABSTRACT

I explore the possibility that the strongly coupled Higgs–fermion sector is responsible for the existence of the fermion generations in a fashion similar to that proposed by Veltman. This appears to be possible in theories in which the vacuum expectation value of the Higgses giving mass to the fermions is small and the corresponding Yukawa coupling strong. An example involving the Higgs triplet is solved in a particular limit which displays an accidental chiral symmetry.

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The attempts to explain the existence of fermion generations fall, roughly speaking, in two categories: those adhering to naturalness as their guiding principle and those attempting to achieve economy. The naturalness has led in turn to supersymmetry, supergravity and superstrings, pushing the scale at which the generations are created all the way to the Planck scale. The minimalist philosophy, on the other hand, envisages a situation much like in nuclear physics, where only two particles, endowed with a strong force, are responsible for hundreds of nuclei. An example of an economic model was suggested by Veltman[1] nine years ago. According to Veltman's proposal, the fermions of the first generation are made up of one basic fermion family bound to a heavy Higgs particle, those of the second generations containing two Higgses and so on. One difficulty with this very attractive scheme is technical—we have no way of dealing with the strong coupling, *i.e.* of computing the masses. On a more fundamental level, the Higgs sector may be nonlinear[2] and the Higgs particle need not exist (similar to the σ in QCD). In fact, for the Higgs masses of the order of several TeV which are needed for the model it may already be impossible to talk about a particle. The principal argument against such a scheme has been that it is unnatural (in much the same way that the natural models are uneconomical) in that there appears to be no symmetry which would make the bound states of massive constituents light.

In this Letter I consider a variant of Veltman's model which has certain amount of calculability and which possesses a limit which might be called natural. It is based on isotriplet rather than the isosinglet Higgs field. In other words, I propose that the fermions of the higher generations are bound states of those of the first generation and the strongly coupled isotriplet Higgs field. Such a situation may occur, for instance, in left right symmetric models[3] where the fermions do not couple to those Higgs fields which are chiefly responsible for the W and Z masses. Consider a model of this kind containing *one* fermion family (e, ν_e, u, d) and the Higgs sector which we shall assume strongly interacting. The fermion masses arise from the Yukawa coupling of the fermions with the Higgs field Φ which is $(\mathbf{2},\mathbf{2})$ under $SU(2)_L \otimes SU(2)_R$ and after the symmetry breaking becomes triplet under the global $SU(2)$ (isospin). The (left and right) W and Z bosons, on the other hand, obtain their masses both from the field Φ and from other $SU(2)_L$ and $SU(2)_R$ Higgs fields Δ_L and Δ_R to which the fermions do not couple. More precisely, the vector boson masses are proportional to $g^2(<\Phi>^2 + <\Delta_{L(R)}>^2)^{1/2}$. Let us suppose now that the small fermion masses originate from very small vacuum expectation value $<\Phi>$, much smaller than $<\Delta_L>$ and $<\Delta_R>$. The Yukawa couplings with Φ must then be large in order to account for the fermion masses. Apart from the weakly coupled gauge boson sector, we thus have a strongly coupled fermion-boson sector.

Consider now the model of generations in which the second generation fermions are bound states of the first generation ones and the triplet Higgs, symbolically

$$\begin{pmatrix} c \\ s \end{pmatrix} \sim \begin{pmatrix} \Phi_1^0 & \Phi_1^+ \\ \Phi_2^- & \Phi_2^0 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}. \tag{1}$$

We now have to address the question of dynamics. The main problem encountered when one attempts to build quarks and leptons from heavy constituents is apparent lack of symmetry which would keep them light. I will show that, in a certain $1/N$ approximation, this model has a chiral symmetry which keeps the bound states much lighter than their heavy scalar constituent. The truncated model which we are able to solve has one bound state (the "second generation") whose mass is

$$M = 2m + \frac{32\pi^2 m_\Phi^2}{G\mu N \ln \frac{\Lambda^2}{m_\Phi^2}}, \tag{2}$$

where m is the mass of the first generation quark, G the Yukawa coupling, μ the cubic scalar coupling and Λ the cutoff. The chiral limit corresponds to $m \to 0$ and $G\mu \to \infty$, that is light first generation *and* strong coupling.

By the large-N limit we shall mean the limit of large global symmetry of the model and not of its gauge group. Thus, neglecting the gauge bosons which are weekly coupled and the Higgs fields which do not couple to the fermions, the Lagrangian of the first generation fermions and the (isotriplet) Higgs field is

$$\mathcal{L} = \overline{\psi}\partial\!\!\!/\psi + G\overline{\psi}\Phi\psi + V(\Phi). \tag{3}$$

As mentioned above, we shall consider the situation where $< \Phi >$ is very small and the Yukawa coupling as well as the scalar self couplings μ and λ large.

It is convenient to consider the isospin breaking separately from the mass splitting among the generations, which is our aim here. Therefore we shall assume that the above Lagrangian has a global $SU(2)$ symmetry under which

$$\psi \to U\psi \quad \text{and} \quad \Phi \to U\Phi U^{\dagger}. \tag{4}$$

It is this picture which we would like to formulate for arbitrary N. We therefore introduce one fermion generation in **N** of $SU(N)$ and one scalar field in the adjoint representation. The second generation created by binding the fermions with the scalars is again an N-plet. This aspect of the model can thus be consistently generalized to arbitrary N.

If we now scale the Yukawa coupling and the cubic scalar coupling as $G^2N = const.$ and $\mu^2 N = const.$ and the quartic coupling as $\lambda N = const.$, it is clear that in the large N limit the planar graphs dominate. In the fermion-scalar channel these are shown in Fig. 1. It is clear that the planar expansion cannot be summed and that further simplifications are necessary in order to extract some information about the spectrum. In case of QCD these consisted of going to two dimensions[4] where a particular choice of gauge effectively eliminates the gluons as dynamical degrees of freedom and one is left with the ladder diagrams which

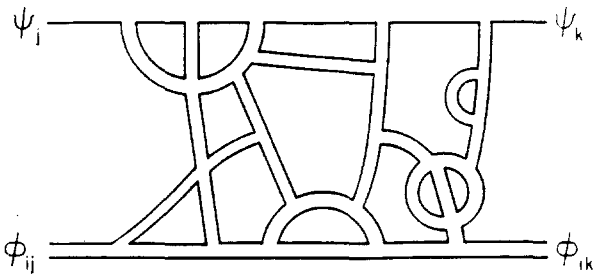

Fig. 1. A planar diagram contributing to Higgs–fermion bound state.

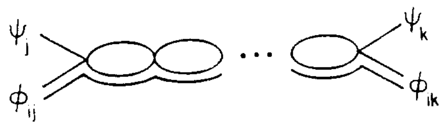

Fig. 2. The bubble diagram.

can be summed. In our model we are able to solve the truncated version consisting only of the ladder diagrams of Fig. 2. This is admittedly a crude approximation and it is not clear to what extent the results are going to be representative of the full theory. The graphs of Fig. 2 are now the leading diagrams of the effective nonrenormalizable theory

$$\mathcal{L} = \overline{\psi^i}(i\partial\!\!\!/ - m)\psi^i - \Phi_{ij}(\partial^2 - m_\Phi^2)\Phi_{ij} - f(\overline{\psi^i}\Phi_{ij}\Phi_{jk}\psi^k + \text{h. c.}), \tag{5}$$

with the coupling constant $f = G\mu/m_\Phi^2$ which scales with N like $fN = const.$ A similar vector model has been studied by Higashijima, Suura and this author, Ref. 5. We shall use here the same method in order to solve the theory (5).

We start by transforming the Lagrangian (5) to an equivalent one which is Gaussian with respect to ψ_L, ψ_R and Φ, by adding the following term to it:

$$-\frac{1}{f}\{(\overline{\Psi^j}_L - f\overline{\psi^i}_L\Phi_{ij})(\Psi^j_R + f\Phi_{jk}\psi^k_R) + \text{h. c.}\}. \tag{6}$$

We can always add such a term to the Lagrangian. The functional integration over Ψ^i_L and Ψ^i_R takes us back to the original Lagrangian, Eq. (5). The equivalent Lagrangian is then

$$\mathcal{L} = \mathcal{L}_0(\psi_L, \psi_R, \Phi) - \frac{1}{f}\overline{\Psi^j}\Psi^j + (\overline{\Psi^j}_L\Phi_{jk}\psi^k_R - \overline{\psi^i}_L\Phi_{ij}\Psi^j_R + \text{h. c.}). \tag{7}$$

Ψ^i_L and Ψ^i_R represent composite operators $\Phi_{ij}\psi^j_L$ and $\Phi_{ij}\psi^j_R$ and the spinor Ψ^i

describes a composite Dirac fermion if its propagator has a pole. The Ψ^i propagator is obtained from the free part of the effective Lagrangian defined by

$$e^{iS_{eff}(\Psi,\overline{\Psi})} = \int \mathcal{D}\Phi_{ij}\mathcal{D}\psi^i_L\mathcal{D}\overline{\psi^i}_L\mathcal{D}\psi^i_R\mathcal{D}\overline{\psi^i}_R \; e^{i\int \mathcal{L}d^4x}. \tag{8}$$

The path integral can now be performed leading to the following result for the free part of S_{eff}:

$$S_0 = -\frac{1}{f}\int d^4x \overline{\Psi^i}\Psi^i + i\mathrm{Tr}\left(\frac{1}{\partial^2 + m_\Phi^2}\overline{\Psi^i}\gamma_5\frac{1}{i\partial\!\!\!/ - m}\gamma_5\Psi^i\right). \tag{9}$$

The Ψ propagator is related to the Fourier transform of this expression and can be written as

$$\begin{aligned} iG^{-1}(p) &= -\frac{1}{f} + N\int \frac{d^4k}{(2\pi)^4 i}\frac{1}{(p-k)^2 - m_\Phi^2}\gamma_5\frac{1}{k\!\!\!/ - m}\gamma_5 \\ &\equiv p\!\!\!/ A - B. \end{aligned} \tag{10}$$

The integral can be evaluated by using the cutoff Λ yielding

$$A = \frac{N}{32\pi^2}\ln\frac{\Lambda^2}{m_\Phi^2} \tag{11}$$

and

$$B = \frac{1}{f} + \frac{Nm}{16\pi^2}\ln\frac{\Lambda^2}{m_\Phi^2}. \tag{12}$$

For the mass of the bound state we thus have

$$M = \frac{B}{A} = 2m + \frac{32\pi^2 m_\Phi^2}{G\mu N \ln\frac{\Lambda^2}{m_\Phi^2}}. \tag{13}$$

Note that the second term gives a contribution to M which goes as m^{-1}. The above result can be understood in terms of an accidental chiral symmetry of the effective theory, Eq. (8).[5] Although the Lagrangian (5) does not have chiral symmetry, the effective Lagrangian defined by Eq. (8) is chiral symmetric in the limit $m \to 0$ and $\frac{m_\Phi^2}{G\mu N} \to 0$.

In conclusion, the particular subset of the planar diagrams which we were able to sum up seem to indicate that the strongly coupled Higgs–fermion sector may possess an approximate chiral symmetry. If this result is physical and not an artifact of the approximation made, it suggests the possibility that the Higgs sector is rich and strongly interacting and ultimately responsible for the existence of the fermion generations. Among the questions which have not been addressed here, probably the most interesting is the one concerning other possible channels where bound states may occur, in particular the fermion-antifermion channel which might give rise to leptoquarks and other exotic particles. Another question is how to incorporate the isospin breaking in the model. These and related questions will be the subject of a future study.

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